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CHARACTERISTICS OF IMAGES DERIVED FROM FACTORABLE POLYNOMIALS

Scripps Institute of Oceanography

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BEFORE COMPLETING FORM 19 REPORT DOCUMENTATION PAGE 2 GOVT ACCESSION NO. RECIPIENT'S CATALOG MIMBER RADC-/TR-81-237 Final Technical Kep RACTERISTICS OF IMAGES DERIVED FROM ACTORABLE POLYNOMIALS - Dec 8 S. CONTRACT OR GRANT NUMBERYS) McGlamery F30602-80-C-0142 PERFORMING ORGANIZATION NAME AND ADDRESS Scripps Institute of Oceanography O. PROGRAM ELEMENT. 63428F Visibility Laboratory 21270105 San Diego CA 92120 11. CONTROLLING OFFICE NAME AND ADDRESS 12. REPORT DATE September 1981 Rome Air Development Center (RADC/OCSE) Griffiss AFB NY 13441 14. MONITORING AGENCY NAME & ADDRESS/If different from Controlling O SECURITY CLASS. (of this report) Same UNCLASSIFIED 154. DECLASSIFICATION/DOWNGRADING N/A SCHEDULE 16. DISTRIBUTION STATEMENT (of this Report) Approved for public release; distribution unlimited. 17. DISTRIBUTION STATEMENT (of the electract entered in Block 20. If different from Report) Same 18. SUPPLEMENTARY NOTES RADC Project Engineer: John J. Vasselli (RADC/OCSE) 9. KEY WORDS (Continue on reverse side if necessary and identify by block number) Image Reconstruction Autocorrelation Functions Ambiguity Speckle Imaging Factorable Polynomials 0. ABSTRACT (Continue on reverse side if necessary and identify by block number) Image reconstruction techniques exist which use as the input the autocorrelation function of the image. The uniqueness of the reconstructed image has been questioned. This report considers a class of images which are ambiguous in that many different images can be generated which have the same autocorrelation function. It is shown that two dimensional images generated from factorable polynomials which have unity power factors are basically uninteresting images. However, by using (Cont'd)

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non-unity power factors, images with structure results. This structure varies significantly among images with the same autocorrelation function. Finally, it is pointed out that this factoring process is equivalent to convolution of the image with a pair of delta functions, leading to the ability of generating ambiguous images from functions that are not completely factorable. Examples are shown. The conclusion of the work is that caution must be exercised in interpretation of images reconstructed from autocorrelation functions.

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CHARACTERISTICS OF IMAGES DERIVED FROM FACTORABLE POLYNOMIALS

INTRODUCTION

In recent years image processing techniques have been developed which use as an input the autocorrelation function of an image or, equivalently, the modulus square of the Fourier transform of the image. Fienup's 1 technique is a good example of this approach. In his technique an iteration procedure is used which alternates between spatial and frequency domains, enforcing known constraints in each domain. In the spatial domain the constraints are non-regativity of the image brightness values and limited spatial extent. In the frequency domain the constraint is that the processed image Fourier transform must have a modulus equal to the original modulus. This processing has had a promising degree of success on the limited number of images processed thus far.

In the absence of the non-negativity constraint there are an infinite number of objects which can produce the same autocorrelation function since the phase component of the Fourier transform can be varied at will without affecting the image autocorrelation function while affecting the image significantly, usually

creating regative image values. The enforcement of non-negativity of the image, however, limits the number of images which have the same autocorrelation function.

An important question is how many non-negative images can produce the same autocorrelation function? There seems to be no general theory for two dimensional There has been some work done on specialized two dimensional images. Fried², following the work of Bruck and $Sodin^3$, has developed a method for developing specialized two dimensional images which have a high degree of ambiguity in the sense that a large number of images can produce the same autocorrelation function and satisfy the non-negative constraint. images derived from completely factorable polynomial functions. Left unanswered in Fried's work was the question of whether such specialized images are of any interest. Are they similar to real world images of interest and are the ambiguous images different from each other in an interesting way? This report addresses these questions.

REVIEW OF THEORY

Let A_{pq} represent the brightness value of an image in p, q coordinates. We define a polynomial function f(u,v) as

$$f(u,v) = \sum_{pq} A_{pq} u^{p_v q}.$$
 (1)

If the polynomial has the characteristics of being completely factorable, then f(u,v) can also be written as

$$f(u,v) = \prod_{r,s} (u + a_r)(v + b_s).$$
 (2)

In Fried's and Bruck and Sodin's work the form used was $(u - a_r)$. The change of sign here is for bookkeeping convenience in computer programs.

The general image $A_{p,q}$ will not have the characteristic of being completely factorable. We are dealing here with special cases. Assuming the image is completely factorable then Equations (1) and (2) can be equated.

$$\sum_{pq} A_{pq} u^{p_{v}q} = \prod_{r,s} (u + a_{r})(v + b_{s}).$$
 (3)

Given the set of (a_r, b_s) the image values A_{pq} are found by multiplying the factors on the right hand side of Eq.3, and equating the coefficients of like powers of u and v. For example let the factorable polynomials be

$$f(u,v) = (u + 1)(u + 3)(v + 2)(v + 4)$$

$$= 24 + 32u + 8u^{2}$$

$$+ 18v + 24uv + 6u^{2}v$$

$$+ 3v^{2} + 4uv^{2} + v^{2}v^{2}.$$
 (4)

The resulting brightness values of the image A_{pq} are:

		p→		
		u^0	u ¹	u^2
q∳	v ⁰	24	32	8
ч¥	v ¹	18	24	6
	v ²	3	4	1

To ensure non-negativity of the image the roots a_r , b_s must be positive reals, or, if complex, occur in complex conjugate pairs with positive real components.

Fried has shown that the autocorrelation function of an image which is completely factorable is not unique, i.e., there are many non-negative images, all completely factorable, which produce the same autocorrelation function. These images are found by replacing any combination of the roots by their reciprocals, a_r^{-1} , b_s^{-1} . For example, for the factors ($u + a_1$)($v + b_1$) the complete set of images would be generated by

$$(u + a_1)(v + b_1)$$

$$(u + a_1^{-1})(v + b_1)$$

$$(u + a_1)(v + b_1^{-1})$$

$$(u + a_1^{-1})(v + b_1^{-1}).$$
(5)

For a polynomial of R u factors and S v factors an image of dimensions (R + 1) by (S + 1) results. By flipping the roots 2^{S+T} images can be generated, all having the same autocorrelation function. One of these is the original, the other its 180° rotation so that $2^{S+T}-2=2^{S+T-1}$ ambiguous images can be formed.

It should be noted that the image developed from this method are very constrained in terms of degrees of freedom. The images are really the product of two one dimensional images, each extended in a direction orthogonal to its variable. Let f(p) be the one dimensional image obtained from the expansion of the u roots and f(q) the one dimensional image obtained from the expansion of the v roots. the two dimensional image f(p,q) is then

$$f(p,q) = f(p) f(q)$$
. (6)

PRELIMINARY IMAGES

A program was written by B. Fahy of the visibility laboratory which allows specification of the u and v factors or roots, which roots are to be flipped, and computes the resulting image, normalized so that the maximum value is 10,000. Results are shown in Tables i to 25 for a variety of root distributions. In all cases the image brightness values decrease monotonically from the maximum value. As the roots increase in value from unity the images become smaller, that is, fall off more rapidly. They also shift towards an edge and become more asymmetrical. Flipping half of the roots changes the location and the symmetry. In general the images produced are uninteresting.

FACTORS AS POINT SPREAD FUNCTIONS

The process of multiplying a polynomial, whose coefficients represent an image, by a factor $(u+a_n)$ is equivalent to convolving that image with a pair of delta functions. For example the polynomial

f

$$f_{n-1}(u) = A_0 + A_1 u + A_2 u^2 + \cdots + A_{n-1} u^{n-1}$$
 (7)

when multiplied by ($u + a_n$) will consist of two components. The first will be the original polynomial with each coefficient multiplied by a_n . The second component will be the original polynomial with the power of each variable increased by one, which corresponds to a right shift of the image associated with the polynomial coefficients. In terms of the image we have convolved the original image with pair of delta functions, one at the origin with a weighting of a_n and the other shifted to the right by one pixel, with a weighting of unity.

If the polynomial is multiplied by $(u^{\delta} + a_n)$ where δ is an integer, the effect on the one-dimensional image associated with the polynomial is to convolve it with a pair of delta functions, now separated by δ pixels. This allows more interesting images to be constructed, that is, images with more structure. In addition, the ambiguous images can be significantly different from one another. This is illustrated in

Fig. 1 which is a graph of two one-dimensional funcitons. The change in going from one image to the other was caused by the flipping of one root.

The convolution concept leads to a less constrained method of generating ambiguous images. It will now be shown that any arbitrary one-dimensional function can be used to produce a pair of ambiguous images by convolving the image with a pair of delta functions. The initial function does not need to be a factorable function.

Let f(p) be a general one dimensional real function. Convolving f(p) with the delta function pair produces a new function g(p), which is the original function weighted by (a) plus the original function shifted by 6.

$$g(p) = a f(p) + f(p-6)$$
 (8)

The autocorrelation function of g(p), C(i), is

$$\Sigma g(p) g(p-i)$$

$$C(i) = \frac{g}{\Sigma g(p)^2}$$
(9)

We substitute Eq.(8) into Eq.(9) and use the relationship of

$$\sum f(p)f(p-i) = \sum f(p-6)f(p-6-i)$$
 (10)

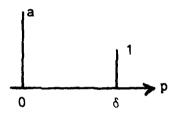
to obtain

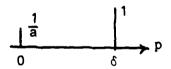
$$C(i) =$$

$$\frac{(1+a^2)\sum f(p)f(p-i) + a\sum [f(p-6)f(p-i) + f(p)f(p-6-i)]}{p}$$

$$\frac{p}{(1+a^2)\sum f^2(p) + 2a\sum f(p)f(p-6)}$$
(11)

If we multiply top and bottom of Eq(11) by $1/a^2$, which leaves C(i) unchanged, the resulting equation is identical to that obtained by replacing a by 1/a. In other words convolving the function f(p) with the two point spread functions





produces images with the same autocorrelation function. Again f(p) can be any function, it need not be derived from a factorable polynomial.

Figure 2 shows a more complex set of ambiguous images. The starting image was an array of seven dots, arranged in the pattern found on a domino. The factor, i.e., the horizontal shifts and weightings used to generate the ambiguous images are shown on the figure. The four images have identical autocorrelation functions. This was verified computationally. The four images have the same basic structure in terms of numbers and locations of dots but the relative brightness of the dots are quite different. The convolving process could be done in the vertical direction to produce even more complex ambiguous images.

DISCUSSION AND CONCLUSIONS

The work shown here has demonstrated that: a) Factorable polynomials using unity power factors, such as $(u + a_n)$, result in smooth images which change little apart from spatial shifts when ambiguous images are generated; b) When greater than unity power factors are used, such as $(u^{y(n)} + a_n)$, structured images can result and the ambiguous images can be quite different from one another; c) The concept of multiplication of a polynomial by a factor is equivalent to convolution of the image represented by the coefficients of the polynomial, with a pair of delta functions. This concept leads to the result that the starting function for generating ambiguous images need not be a factorable polynomial but can be any non negative function. That is, convolution of any starting function with appropriate pairs of delta functions can lead to different images which have the same autocorrelation function. An example of four ambiguous images using the technique, shown in Fig. 2, shows that while basic shape and structure was the same for all images, considerable variation in the internal brightness values occurred between the images.

Images which are ambiguous are very specialized, at least as generated by this technique. Such images may not occur frequently in practice. However, if

image reconstruction is made from the autocorrelation function, the possibility of the reconstructed image being ambiguous should be considered.

At the present there apparently is no theory available which allows determination of whether a given autocorrelation function is unambiguous, that is, whether the autocorrelation function could have been produced by only one non negative image. Instead, the theory deals with the reverse of the problem: methods of generating images which have either ambiguous or unambiguous autocorrelation functions.

Perhaps the approach for now is to rely on the reconstruction techniques themselves for answer to the question of uniqueness. For a wide class of images, with and without noise, do the techniques provide reconstruction of the image within limits set by resolution of the optical system and sensor noise? Comparisons should be made to other image processing approaches.

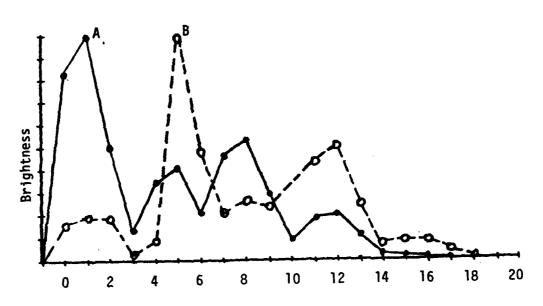


Figure 1. Two factorable functions having the same autocorrelation

function.

A factors: $(u+1)^5 (u^4+5)(u^4+5)(u^7+2)$ B factors: $(u+1)^5 (u^4+5)(u^4+.2)(u^7+2)$

flipped

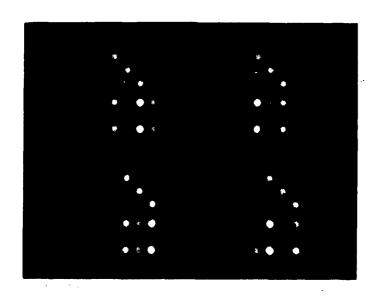
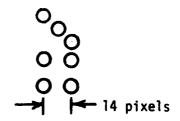


Figure 2. Ambiguous images having the same autocorrelation function. Starting function was a pattern of seven dots arranged as on a domino. Let f(u,v) be the polynomial (not factorable) representing the starting dot pattern. The images are:

Top Left : $f(u,v)(u^7+2)(u^{14}+2)$ Top Right : $f(u,v)(u^7+2)(u^{14}+.5)$ Bottom Left : $f(u,v)(u^7+.5)(u^{14}+2)$ Bottom Right: $f(u,v)(u^7+.5)(u^{14}+.5)$.

Original pattern:



TABLES 1 to 25

These tables represent the brightness values of two dimensional images made from completely factorable polynomials whose factors are of unity power.

TABLE 1. All roots = 1.

	į				:								;	
		-		18	35	39	32	18	Q	-	•	1.0000	1.0000	
	-	15	70	188	330	396	330	188	2	15		1. 0000	1. 0000	!
	9	2	916	820	1488	1785	1488	830	318	70		1. 00001: 0000	1.0000	, e
	18	188	850	2267	3968	4762	3968	2267	028	188	- 18	1. 0000	1. 0000	
	32	330	-1488-	3968	-6945	8334	6945	3968	1488	330	සි -	0000	1. 0000	! ↔
	39	396	1785	4762	- 8334	10000	-8334	4762	1785	396	66	-1-0000 · 1. 0000 ··· 1. 0000	1. 0000 1	
1	35	330	1488	3968	6945	8334	6945	3968	1488	330	35	901. (j	MAGE
	18	188	850	2267	3968	4762	8966	2267	850	188			0 1.0000	- END IMAGE
IMAGE#	9	70	318	820	1468	1785	1468-	850	918	70	•	1: 0000	1. 0000	
) READ	+	13	70	188	939	396	939	188	70	15	**	= -1 : 6000	1. 0000	
ABOUT TO READ IM		-	9	18	35	39	36	18	9	1	θ	URDOTS =	VR0075 =	1

TABLE 2. All roots = 2.

41 - 7 - 0 - 0 - 0	208 39 4 0	9 6 81	625 117 13 0	346 102 11 0	328 61 6 0	136 25 6	39 7 0 0	0 0 1 1	0 0 0	0 0 0 0	2.0000 2.0000 2.0000 2.0000	2.0000 2.0000 2.0000 2.0000	
145	729	1640	2187	1914	1148	874	136	25	a	0	2:-00002	2. 0000 2	•
350	1750	3937	5250	+264	2756	1148	328	10	4		2.0000 2.	2. 0000 2.	
583	2916	6969	8750	7657	4564	1914	346	102	11	•	1		
999	3333	7500	10000	8750	5250	2187	623	113	13		2. 0000 2. 0000	2. 0000	1
909	2500	8298	7500	6999	3937	1640	468	48	o			2, 0000	
222	1111	2500	3333	2916	1750	729	208	39	. 4	•	= 2.0000	2.0000	
*	222	906	999	683	320	145	4	*	0		URDOTS 2. 9999	VR001S 2.0000	

TABLE 3.

1/2 of roots = 2.

1/2 of roots = 1/2.

Compare to TABLE 2.

	c	•	16	56	16	0	CH		0
• •0	8 0	109	208	257	208	109	33	4	0
(1) (1)	193	146	1126	1391	1126	591	193	35	ų
109		1807	3442	4251	3442	1807	591	109	Ø
208		3445	6556	- 6097	6556	3442-	1126	808	16
257		4251	8097	10000	8097	4251	1391	257	50
208	1126	3442	-6556	-8097	9229	3442	1126	508	16
109	591	1807	3442	4251	3442	1807	591	109	œ
35	661	165	1126	1391	1126	165	193	66	2
9	33	109	208	257	208	109	33	9	0
O	C)	8	16	50	16	8	d	0	0
-5999	0.5000	ì	00 0:5	-0, 5000 - 0, 5000 2, 0000 2, 0000	- 0000	2. 0000	20000	2-0000-2-0000	2.0000
0. 5000	0.5000	0. 5000	i	0.5000 2.	2.0000	2.0000	2.0000	2. 0000	2. 0000
		FND IMAGE	MAGE		+	1	1	1	

TABLE 4. All roots = 4.

L TUDBY	ABOUT TO READ	TUPEL *		•						
4964	9160	3555	2370	1037	-116-	. +9	6		•	
3160	7902	8889	5926	2592	777	162	23	N	0	0
3555	6889	10000	5999	2916	875	182	26	Q1		•
2370	5926		4444	1944	583	121	17	-	0	0
1691	- 2592	2916	1944	850	255	53	-4	•		
311	777	875	583	255	76	15	CI	0	0	0
64	162	182	121	53	15	3	0			0
0	23	26	17	^	Ct	٥	0	٥	0	0
	7	2	-	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
•		•		0	0	0	0			•
UR0015	# 0000 ·		4.00	900 4. (9000 4	0000	4.0000	4.0000 - 4.0000 - 4.0000 - 4.0000 - 4.0000 - 4.0000	4.0000	4.0000
VR001S 4. 0000	4. 0000	4. 0000	4. 0000	į	4. 0000 4	4.0000	4 0000	4.0000	4. 0000	4. 0000
			- END IMAGE	MAGE		-4		1	!	

TABLE 5.

1/2 of roots = 4, 1/2 of roots = 1/4. Compare to TABLE 4.

•	0 0	0 11 10			4.0000	
	0	19	61 4 6		4,0000	
	4 6	170 435 608	170	4 0	4.0000	
	19	781 2002 2796	2002 781 170	19	0.2500 0.2500 0.2500 4.0000 4.0000 4.0000 4.0000 4.0000	
1	435	2002 5126 7159	5126 2002 435	49	4. 0000	
	69	2796 7159	7159 2796 608	69	-2500	
1	495	2002 5126 7159	5126 2002 435	49	2500 0	END IMAGE
	19	781 2002		19	500 0.	0, 2500 0.
IMAGE#	4 %	170	170	4	!	_ !
ABOUT TO READ		19	6 6 7	0	3 = 0.2500 5 = 0.2500	0 0 2500
ABOUT 1	0			0	URGOTS 0.2500	0.2500

TABLE 6. All roots ≈ 8.

													!	
	•	0	0	0	•	0	0	0	0	0	0	B. 0000	B. 0000	
	0	0	0	0	0	0	0	0	•	0	0	B. 0000	B. 0000	
	6	°o	0	0	0	0	0	0	- 0	0		8-0000 8-0000	8. 0000	
	0	0	0	0	0	٥		0	•	0		-8: 6006 8: 0000 8: 6000	B. 0000	1
-	#P	4	6	#	0	0	•	0		0	0	. 0000	B. 0000	i H
	64	61	34	11	d	٥	•	0	0	0	0	9000	B. 0000 B	
1	328	410	530	76	. 16	C)	0	0	0	0		i .	,	END IMAGE
	1500	1875	1054	351	76	11	+	0	•	0	•	- 0000 - 8- 0000	00 B. 0000	END
IMAGE#	4500	5625	3164	1054	230	34	6	0	0	0			8 0000	
ABOUT TO READ IMA	- 0008	10000	5625	1875	410	61	9	0	0	0	•	= 	B. 0000	-
ABOUT	-6400	8000	4500	1500	328	49		0		0		URDOTS B. 9999	VR001S 8. 0000	
													1	

TABLE 7.

1/2 of roots = 8.
1/2 of roots = 1/8.
Compare to TABLE 6.

0 0 1 4 8 4 1 0 0 2 17 73 143 73 17 2 1 17 141 610 1191 610 141 17 4 73 610 2629 5127 2629 610 73 8 143 1191 5127 10000 5127 1191 143 1 17 141 610 1191 610 141 17 0 0 0 1 4 8 4 1 0		•	(•		•					
1 4 8 4 1 2 17 73 143 73 17 17 141 610 1191 610 141 73 610 2629 5127 2629 610 143 1191 5127 10000 5127 1191 73 610 2629 5127 2629 610 73 610 2629 5127 1191 17 141 610 1191 610 141 2 17 73 143 17 2 17 73 143 17 0 1 4 8 4 1 0 0 0 0 0 0		•		•	•	Φ	0	θ	-		•
17 73 143 73 141 610 1191 610 141 73 610 2629 5127 2629 610 143 1191 5127 10000 5127 1191 73 610 2629 5127 1191 73 610 2629 5127 1191 17 141 610 1191 610 141 2 17 73 143 73 17 0 1 4 8 4 1 0 0 0 0 0 0	0	0	0	-	4	α,	*	=	0	0	0
17 141 610 1191 610 141 73 610 2629 5127 2629 610 73 610 2629 5127 1191 17 141 610 1191 610 141 2 17 73 143 141 2 17 73 143 17 3 17 73 143 73 17 4 8 4 1 0 0 0 0 0 0			C	17	22	143	73	17			•
73 610 2629 5127 2629 610 143 1191 5127 10000 5127 1191 73 610 2629 5127 2629 610 17 141 610 1191 610 141 2 17 73 143 73 17 0 1 4 8 4 1	0	-	17	141	610	1191	610	141	17	-	0
143 1191 5127 10000 5127 1191 73 610 2629 5127 2629 610 17 141 610 1191 610 141 2 17 73 143 73 17 0 1 4 8 4 1 0 0 0 0 0 0		+	73	019	-5655	5127	5629	919		+	
73 610 2629 5127 2629 610 17 141 610 1191 610 141 2 17 73 143 73 17 0 1 4 8 4 1 0 0 0 0 0 0	0	00			5127	10000	5127	1191	143	80	0
17 141 610 1191 610 141 2 17 73 143 73 17 0 1 4 8 4 1		+	7.9		-6893	5127	-595	610	2		•
0 1 4 B	0	-	17	141	610	1191	610	141	17	-	0
0 1 4		•	4		2	143	73	11	*	•	•
\$	0	0	0	~	4	60	4	-	0	0	٥
			•	•		•	0	0		0	
) =	UROOTS =	0-1250	0.1250		66		0000	B. 0000	- B 0000	- B. 0000	B . 000
0.1250 0.1250 0.1250 0.1250 0.1250 8.0000 8.0000	VR00TS = 0. 1250	o ·	0. 1250					B. 0000	8.0000	B. 0000	B. 0000

TABLE 8.

All roots = 16.

6250 3706 10787 292 32 2 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0	3906 1787 292 32 2 0 0 0 0 0 0 0 0 0 1098 183 20 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0	ABOUT TO READ	ro READ	* 4 5 5 5 5		4						
3906 1098 183 20 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0	3906 1098 183 20 1 0 0 0 0 0 0 0 0 15.0000 15.	10000	-6529	1787	262	35	(Li	0		0		
183 51 8 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0	183 51 8 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0	6250	3906	1098	183	50	-	0	0	0	0	0
183 51 8 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0	183 51 8 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0	1787	1098	309	51	5	•	•				
20 5 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0	20 5 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0	292	183	51	œ	0	0	0	0	0	0	0
1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0	35	20	S		0	0	0	•	•		•
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0	0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0	CI	 1	0	ο,	0	0	0	0	0	0	0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0	0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0		0	•	•		0	0				
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0	0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0	0	٥	0	0	0	0	0	0	0	0	0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0	0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0		•		•	•	•		•	•		•
= 16. 0000 1	= 16. 0000 1	0	0	0	0	0	0	0	0	0	0	0
16. 0000 16.	16. 0000 16.	•	•	0	•	•	0	0	0	0		•
16.0000 16.0000 16.0000 16.0000 16.0000 16.0000	16. 0000 16. 0000 16. 0000 16. 0000 16. 0000 16. 0000 16. 0000	UR0015 16: 0000			<u> 16. 0006</u>	16.000	0~16. 0(300 -16	-0000-16	1 0000	6. 0000	16. 0000
	1	VR001S 6. 0000		0 16. 0000	16. 0000		16.		0000	0000		16. 0000

TABLE 9.

1/2 of roots = 16,
1/2 of roots = 1/16.
Compare to TABLE 8.

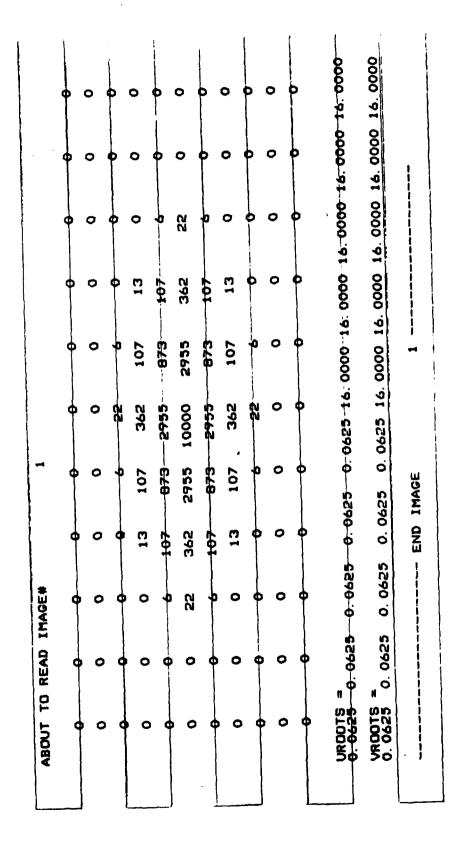


TABLE 10. All roots = 32.

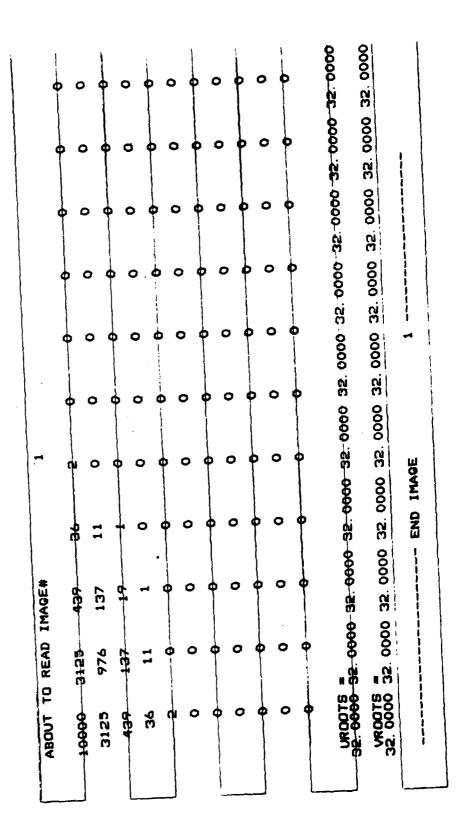


TABLE 11.

1/2 of roots = 32.
1/2 of roots = 1/32.
Compare to TABLE 10.

0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0		
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0	000000	
0 0 0 14 95 0 14 237 1540 0 2 95 1540 10000 0 0 14 237 1540		
0 0 0 14 95 0 2 95 1540 10000 0 0 0 14 237 1540 0 0 0 14 95	00000	
0 2 95 1540 10000 0 2 95 1540 10000 0 14 237 1540	0 0 0 0	
0 2 95 1540 10000 0 14 237 1540 0 0 0 14 95	2000	0 0
0 0 14 95		0
0 0 0 14 95		0
	0 0 0	
0 0 0 0 0 0	0 0 0	0
0 0 0 0 0 0 0 0 0	0	
URDOTS # 6: 6313 6: 6313 6: 6313 6: 0313 -32: 0000 -32: 0000 -32: 0000 -32: 0000	0000 32:000 0 32:00 0	0 32.000(
VRDDTS = 0.0313 0.0313 0.0313 0.0313 32.0000 32.	0. 0313 32. 0000 32. 0000 32. 0000 32. 0000	0 32. 000
1 1		

TABLE 12. All roots = 64.

ABOUT TO READ	READ	IMAGE#		-		,						
10000	1562	109	•	0	9		•	0		•		0
1562	244	17	0	0	•		0	0	0	0		0
109	17	+	0	•			•	•	•	•		•
4	0	0	0	0	0		0	0	0	0	•	0
•	•	•	·	•				•	•	•		•
0	0	0	0	0	0		0	0	0	0		0
•	•	•		•	•			0		•		•
0	0	0	0	0	0		0	0	0	0		0
	•	•	•	þ	0			P	0	•		0
0	0	0	0	0	0		0	0	0	0		0
	•							•				•
URDOTS = 64.0000 64	= 64 : 0000	-64:-0006	-64:0000-64:0000-64:0000	9-64:		4. 0000	64	64. 0000 64. 0000 64. 0000 64. 0000 64. 0000	-0000	-64-000	70 64	000
VRDDTS = 64. 0000 64	± 64. 0000	64. 0000	64. 0000		64. 0000 6	64. 0000		64. 0000 64	64.0000	64. 0000	1	64. 0000
		1	- END IM	IMAGE		-	1	: : : :	1	1		

TABLE 13.

1/2 of roots = 64. 1/2 of roots = 1/64. Compare to TABLE 12.

•		•	0	0	0	0	0	0		
٥	٥	0	0	0	0	0	0	0	0	0
•	•	0		9	•	0	•	•	•	0
0	0	0	0	-	24	***	0	0	0	0
		•	+	99	178	99		0	0	•
0	0	0	42	778	10000	778	24	0	o	0
	•	•	+	99	178	99	-	0	0	0
0	0	0	0	~	24	~	0	0	0	0
•	-0	•	0	•	0	•	0	0	0	6
٥	0	0	o	0	0	0	0	٥	٥	0
0	-		•	•	0	•	0	0	0	P
URDOTS 0. 0156	= -0.0156	0.0156	-0.0156	Ö	-0:01560:0156-64:0000-64.0000-64:0000-64:0000-64:0000	000064.	0000	-0000	4. 0000	64. OC
VRD0TS 0. 0156	= 0.0156	0.0156	0.0156		0.0156 64.	0000 64.	64. 0000 64. 0000 64. 0000 64. 0000 64. 0000	. 0000	4. 0000	64.00

TABLE 14.

Linearly increasing roots, 1 to 10.

אפטרו וט אבאני		IMAGE#								
809 23	2371	2845	1876	762	201	35	4	0	0	0
2371 69	5946	8334	5495 2	2233	589	103	11	0	0	0
2845 83	8334	10000	6594 2	2679	707	123	14	1	0	0
1876 54	5495	6594 4	4348 1	1766	466	81	6	0	.0	0
762 22	2233	2679	1766	717	189	33	m	0	0	0
201 5	589	707	466	189	50	æ	1	0	0	o
35 1	103	123	81	33	8	1	0	0	٥	0
4	11	14	6	က	1	0	0	0	0	٥
0	0	1	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	•	0
URODTS = 1. 0000 2.	2. 0000	3. 0000	4. 0000	5. 0000	1	6. 0000	7. 0000	B. 0000	9. 0000	9. 0000 10. 0000
VRD0TS =	2: 0000	3.0000	4.0000	3.0000		6. 0000	7.0000	B. 0000	9. 0000	10.000
		1	END IMAGE	ii ii		. 		; ; ;] 	

TABLE 15.

Linearly increasing roots, 1 to 10, first half flipped. Compare to TABLE 14.

:		1		i		į		i		- 1		ì	İ	Ì
	0	0	0	0	0	0	0	0	0	٥	0	9. 0000 10. 0000	10:0000	
	0	0	0	a	m	a	-	0	0	٥	•	9. 0000	9: 0000 10: 0000	1
	0	CI	14	43	99	51	19	ဗ	o `	٥	0	B. 0000	8.0000	
	1	21	130	388	297	462	175	35	က	0	0	7. 0000	9.3333 6.25060.2000-4: 00007: 00008: 0000	
	9	106	643	1916	2943	2279	998	175	19		0	6. 0000 7. 0000	0000	-
	17	279	1694	5044	-7747	6001	2279	462	51	N	0	0. 2000 6	2000-	
1	23	360	2187	6512	10000	7747	2943	297	99	m	0	- 1	.0 000	END IMAGE
	15	234	1424	4240	6512	5044	1916	388	43	a	0	3 0.2500	3-0-6	- END
MAGE#	ស	78	478	1424	2187	1694	643	130	14	0	0	0. 3333	0. 333	1
READ 1	0	13	78	234	360	279	106	21	C	0	0	0. 2000	= -0. 5000	
ABOUT TO READ	0	0	•	15	23	17	9	•	0	0	0	URBBTS-1. 0000	VR001S =	1
											}			

TABLE 16.

Linearly increasing roots, 1 to 10, second half flipped. Compare to TABLE 14.

ABOUT TO	READ	IMAGE#								
0	0	0	0	0	0	٥	0	٥	٥	0
0	0	0	0	~	ຎ	n	CI	0	0	٥
0	0	0	e.	19	51	99	43	14	Cu	o .
0	Ö	ო	33	175	462	597	388	130	21	-
0	-	19	175	866	2279	2943	1916	643	106	4
0	N	51	462	2279	6001	7747	5044	1694	279	17
0	6	99	597	2943	7747	10000	6512	2187	360	53
0	U	43	388	1916	5044	6512	4240	1424	234	15
0	0	14	130	643	1694	2187	1424	478	78	n
0	0	CI	21	106	279	360	234	78	13	0
0	0	o		9	17	23	15	'n	0	0
URBBTS -	2. 0000	3.0000	4. 0000	J	5. 0000 0	0. 1667	0.1429	0. 1250	0. 1111	0.1000
VR0015 =	3000 ∹	3.0000	4. 6060		5.0000 0	0.1557	0.1429	0.1230	0.1111	0. 1000
				!		•			i i	

TABLE 17.

Linearly increasing roots, 10 to 100.

10000 2929 351 23 0 <th< th=""><th></th><th></th><th></th><th></th><th></th><th></th><th></th><th></th><th></th><th></th></th<>										
857 102 6 0 <td></td> <td> 6</td> <td>23</td> <td>0</td> <td>0</td> <td>0</td> <td>0</td> <td>0</td> <td>0</td> <td>0</td>		6	23	0	0	0	0	0	0	0
102 12 0	1		•	0	0	0	0	0	0	0
			0	0	0	0	0	0	0	0
			0	0	0	0	0	0	0	0
			0	0	0	0	0	0	0	0
			0	0	0	0	0	0	0	0
			0	0	0	0	0	0	0	0
			c	0	0	0	0	0	0	0
0 0 0 0 0 0 0 0 0			0	0	0	0	0	0	0	0
			0	0	0	0	0	0	0	0
			0	0	0	0	0	0	, O	0

10. 0000 20. 0000 30. 0000 40. 0000 50. 0000 60. 0000 70. 0000 80

- END IMAGE

TABLE 18.

Linearly increasing roots, 10 to 100, first half flipped. Compare to TABLE 17.

	o	0	0	0	0	0	0 0	0 0 0	0 0 0	00000
	٥	0	0	0	0	0	0 0	0 0 0	0000	0 0 0 0
i	0	0	0	0	0	0	0 0	0 0 0	0000	0 0 0 0
!	0	0	0	0	8	16	16	16	9 - 0 0	1 0 0 0
	0	0	0	11	144	640	640	40	40 40 0	640 40 1 0
,	o	0	9	185	2982	10000	10000	10000 640 16	10000 640 16	10000 640 16 0
•	٥	0	1	41	511	2562	2262	144	144	144 144 3 0
	0	0	0	ო	41	185	185	185	111 0	111 0 0
IMAGE#	٥	0	0	0	1	9	0	0 0	0 0 0	10000
READ	0	0	0	0	0	0	0 0	0 0	0000	0 0 0 0
ABOUT TO	0	0	0	0	o	0	0 0	0 0 0	0 0 0 0	0 0 0 0

END IMAGE

TABLE 19.

second half flipped. Compare to TABLE 17. Linearly increasing roots, 10 to 100,

1		- 1		- 1				- 1		(1	4	
	0	0	0	0	0	0	0	۵	0	0	٥	0.0010	0:00:0	
	0	0	0	0	0	0	٥	0	0	0	0	0.0011	0.0011	1
	0	0	0	0	0	7	**	0	0	0	0	0.0125	0.0123	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
	0	٥	0	0	60	185	42	Ð	O	0	0	0. 0143	0.0143	
	0	0	٥		102	2268	514	42	-	0	0	0.0167	0167	-
;	0	0	0	7	452	10000	2268	185	7	0	0	50.0000 0	0000	
7	0	0	0	0	20	452	102	œ	0	0	0	ţ	500	IMAGE
	0	0	a	0	0	7	1	0	0	0	0	40.0000	40.000	END IM
IMAGE#	o	0	0	0	o	0	0	٥	o	0	0	30.0000	30. 6000	3 3 4 5
	0	0	0	0	0	0	0	0	٥	0	0	20. 0000	20. 0000	1
ABOUT TO READ	0	0	o	0	0	0	0	0	0	0	0	10. 0000 2	VRDDTS = 10.0000 2	, , , ,
	1		}		}		1		1		Į.	}	}	

TABLE 20.

Exponentially increasing roots, 12 steps.

	!	1		-		GE	END IMAGE		1	
22. 6274	18.0000	1	8.0000-11.3137	J. 6358 8	4.0000 5.	ļ	2.8284	2.0000	1.4142	VR0015
22. 6274	16. 0000 8	11. 3137	B. 0000 11	5. 6568 8	4. 0000 5.		2. 8284	2. 0000	1. 4142	URBUTS 1. 0000
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	٥	0	٥	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	-	4	10	14	10	m
0	0	0	0	H	11	42	66	137	101	30
0	0	0	1	11	88	258	909	828	612	185
0	0	0	4	42	258	971	2259	3116	2305	269
0	0	0	10	66	909	2259	5258	7251	5364	1622
0	0	0	14	137	828	3116	7251	10000	7398 1	2236
0	0	0	10	101	612	2305	5364	7398	5473	1654
0	0	0	m	30	185	269	1622	2236	1654	200

TABLE 21.

Exponentially increasing roots, 12 steps, first half flipped. Compare to TABLE 20.

ABOUT TO READ	TO R		IMAGE#								
0		n	26	59	2	5.4	12	1	0	0	0
'n		89	311	200	827	501	146	21	1	0	0
58	, c	311	1415	3186	3762	2280	665	66	7	0	0
59	7	700	3186	7172	8469	5132	1497	224	17	0	0
20	00	827	3762	8469	10000	0909	1767	264	20	0	0
45	ij	501	2280	5132	0909	3672	1071	160	12	0	٥
12	-	146	665	1497	1767	1071	312	46	ຕ	0	0
-		21	66	224	264	160	46	7	0	0	0
0		-	7	17	50	12	9	0	0	0	0
0		0	0	0	0	0	0	0	0	0	0
0		0	0	0	0	0	0	0	,	0	0
URBBTS 1. 0000	Ţ	0. 7071	0. 5000	0.3535	- }	0. 2500 5.	5. 6568	B. 0000	11. 3137	16. 0000	22. 6274
VR001S	H 6	7071	0: 5000		0.35350:2500-		5. 6568	B. 0000	11:3137	B. 0000 11:3137 15:0000 22:627	22. 62
 	 	i 1	 	FND IMAGE	AAGE		-	1	1	# # #	

TABLE 22.

Exponentially increasing roots, $\sqrt{2}$ steps, second half flipped. Compare to TABLE 20.

				1						I			- 1
	o	0	0	1	12	42	70	59	26	32	0	0.0442	0.0442
	0	0	7	21	146	501	827	700	311	89	ກ	0.0625	0.0623
	0	0	7	66	665	2280	3762	3186	1415	311	56	0.0884	0.0884
	0	0	17	224	1497	5132	8469	7172	3186	700	29	0. 1250	0.1250
	0	0	20	264	1767	9099	10000	8469	3762	827	70	0.1768	2.8284 4.0000 -0.1768 0.1250
	0	0	12	160	1071	3672	9909	5132	2280	201	42	4. 0000 0	00000
-	0	0	e	46	312	1071	1767	1497	665	146	12	Ì	40
	0	0.	0	7	46	160	264	224	66	21	-	2.8284	- }
MAGE#	,0	0	0	0	n	12	20	17	7		0	2. 0000	-2-0000
READ I	0	0	0	0	0	0	0	0	0	0	0	1. 4142	4 4442
ABOUT TO	0	0	0	0	0	0	0	٥	0	0	0	URBBTS -	VR00TS =
	i		1		i		!		•		1		l

END IMAGE

TABLE 23.

Exponentially increasing roots, x2 steps.

ABOUT	ABOUT TO READ	IMAGE#		-					1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1	
	800	3330	947	125	~	0	0	0	0	0
		2000		250	15	0	0	0	0	0
2002	20001	7000		146	10	0	0	0	0	0
3330	6634	144		1 6	, (c	0	0	0	0
947	1893	1260	308	*	,	•		•	c	o
125	250	166	47	•	٥	0	>	•	•	• (
7	15	10	n	0	0	0	0	0	0	
	C	0	0	0	0	0	٥	0	0	0
		· c	0	0	0	0	0	0	0	0
>	•	, ,		0	0	0	0	O	0	0
0	0	>	•	•	•	(<	C	0	0
0	0	0	0	0	2	>				
	0	0	o	0	0	0	0	0	0	o
						- 1		7	7404	00 6159
1.0000	2 2 000	0000 4 0000	O 8 0000	16.0000	90 32.	9000	64. 0000	\$1 ₹8. 00	0 0 0 14	
VRODTS 1: 0000	S = 2: 000	900 - 000	0000 B 0	16. 0006	32.	1 0000	54. 0000	• 128: Q	0 1256:00	*51E: 00
1	1 1 1		- END IMAGE	IGE			1			

TABLE 24.

Exponentially increasing roots, x2 steps. First half flipped. Compare to TABLE 23.

		}				j		}		1			1	J
	0	0	0	0	0	0	0	0	0		0	\$512.00	15 OC	
			•			Ì						\$51	104	
i	0	0	0	0	0	0	0	0	0	0	0	\$256.00	9 3	
,												\$256	\$25(
	0	0	0	0	0	0	0	0	0	0	٥	00	8	
) (\$128.00	\$128.00 \$256.00 \$512.00	1
	0	0	0	C	n	m	0	0	0	0	٥	64. 0000	64. 0000	
	0	4	_	_	OI.	m	6	0	0	0	٥	- 1		
			47	191	312	173				_		32. 0000	0000	
	u	83	842	00	63	46	173	ო	0	0	0	(છો 	
			Ď	3400	5563	3094	1					0.0625	-6-12500: 0625	
•	4	150	1514	6112	10000	5563	312	N)	0	0	0	1	0	GE
			-	\$	10	B)		,				0. 1250	1250	IMAGE
	G	92	925	3736	6112	3400	191	m	0	0	0	0	0	END
_				C.	•	m						200	909	1
IMAGE#	0	22	555	925	1514	842	47	0	0	0	0	0. 2500	0.2500	
	į				1		}		}			8	1	
REAI	0	CV	22	42	150	83	4	0	0	0	0	0. 5000	-5900	
10			}				{	-				•	"	1
ABOUT TO READ	0	0	0	a	4	CI	0	0	0	0	0	URBBTS 1. 0000	VRDD15	1 1
							}				}			

TABLE 25.

Exponentially increasing roots, x2 steps. second half flipped. Compare to TABLE 23.

,		:								1			1	
	0	0	٥	0	0	2	4	2	0	0	0	0.0019	0.0019	
	٥	0	٥	0	4	83	150	92	22	2	0	0.0039	0.0039	:
	0	0	0	0	47	842	1514	925	229	23	0	0.0078	0.007B	
	0	0	٥	က	191	3400	6112	3736	925	92	(N	0.0156	0.0136	
	0	0	٥	5	312	5563	10000	6112	1514	150	4	0.0313	0313	-
	0	0	0	3	173	3094	2563	3400	842	83	61	l	9000	
-	0	0	0	0	6	173	312	191	47	4	0	16. 0	16. 0000	LLI C
	0	0	0	0	0	m	n	8	0	0	0	8. 0000 16. 0000	B. 0000	END IMAGE
IMAGE#	0	0	0	0	0	0	0	0	0	٥	0	4. 0000	4. 0000	[]]]
۵	0	0	0	0	0	٥	0	0	0	0	0	2. 0000	2. 0000	; ; ; ;
ABOUT TO REA	0	0	0	0	0	0	0	0	0	0	0	URBBTS 1. 0000	VR001S =	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
	1		l		l		1		[Ι.	1	1	

REFERENCES

- J.R. Fienup, "Reconstruction of an Object from the Modulus of its Fourier Transform" Optics Letters, Vol. 3, p. 27, July 1978.
- David L. Fried, "Ambiguity in Image Extraction from Speckle Measurements Using Non-negativity", Report TR-369, Optical Sciences Company, P.O. Box 446, Placentia, Ca. 92670, March 1980.
- 3. Yu. M. Bruck and L.G. Sodin, "On the Ambiguity of the Image Reconstruction Problem", Optics Communication, Vol. 30, Number 3, September 1979.